

# Pre-calculus Summer Packet Solutions

## \*PART 1

$$\textcircled{1} -3x^3 + 27x$$

$$-3x(x^2 - 9)$$

$$-3x(x+3)(x-3)$$

$$\textcircled{2} 2x^3 - 16$$

$$2(x^3 - 8) \leftarrow \text{difference of cubes}$$

$$2(x-2)(x^2 + 2x + 4)$$

$$\textcircled{3} x^2 + 10x + 16$$

$$(x+8)(x+2)$$

$$\textcircled{4} x^4 + 3x^2 + 2$$

$$(x^2 + 2)(x^2 + 1)$$

$$\textcircled{5} x^4 - 5x^2 + 4$$

$$(x^2 - 4)(x^2 - 1)$$

$$(x+2)(x-2)(x+1)(x-1)$$

$$\textcircled{6} (-2x^4 + x^3) + (6x + 3)$$

$$-x^3(2x+1) + 3(2x+1)$$

$$(-x^3 + 3)(2x+1)$$

$$\textcircled{7} (x^5 + 4x^3) + (x^2 - 4)$$

$$x^3(x^2 + 4) - 1(x^2 + 4)$$

$$(x^3 - 1)(x^2 + 4)$$

$$(x-1)(x^2 + x + 1)(x^2 + 4)$$

$$\textcircled{8} 2x^6 + 3x^3 - 2$$

$$(2x^3 - 1)(x^3 + 2)$$

$$\textcircled{9} x^6 - 1$$

$$(x^3 + 1)(x^3 - 1)$$

$$(x+1)(x^2 + x + 1)(x-1)(x^2 - x + 1)$$

$$\textcircled{10} -2x^5 + 18x^3 - 16x^2 + 144$$

$$-2[x^5 - 9x^3 + 8x^2 - 72]$$

$$-2[x^3(x^2 - 9) + 8(x^2 - 9)]$$

$$-2(x^3 + 8)(x^2 - 9)$$

$$-2(x+2)(x^2 - 2x + 4)(x+3)(x-3)$$

## \*PART 2

$$\textcircled{1} (x-1)(x+1) = 0$$

$$x-1=0, x+1=0$$

$$x=1, x=-1$$

$$\textcircled{2} (2x-3)(4x+1) = 0$$

$$2x-3=0, 4x+1=0$$

$$x = \frac{3}{2}, x = -\frac{1}{4}$$

$$\textcircled{3} x^2 - 36 = 0$$

$$(x+6)(x-6) = 0$$

$$x+6=0, x-6=0$$

$$x = -6, x = 6$$

(continued)

## \* PART 2 (Continued)

$$\begin{aligned} \textcircled{4} \quad x^2 + 5x &= -6 \\ +6 \quad +6 \\ \hline x^2 + 5x + 6 &= 0 \\ (x+3)(x+2) &= 0 \\ x+3=0, x+2=0 \\ \hline x &= -3, x = -2 \end{aligned}$$

## PART 3

$$\begin{aligned} \textcircled{1} \quad f(5) &= 5^2 + 2(5) + 3 \\ &= 25 + 10 + 3 \\ &= \boxed{38} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad g(-1) &= (-1)^2 - 1 \\ &= 1 - 1 \\ &= \boxed{0} \end{aligned}$$

$$\textcircled{3} \quad f(\text{banana}) = \boxed{\text{banana}^2 + 2\text{banana} + 3}$$

$$\begin{aligned} \textcircled{4} \quad f(x-3) &= (x-3)^2 + 2(x-3) + 3 \\ &= x^2 - 6x + 9 + 2x - 6 + 3 \\ &= \boxed{x^2 - 4x + 6} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad g(x+5) &= (x+5)^2 - 1 \\ &= x^2 + 10x + 25 - 1 \\ &= \boxed{x^2 + 10x + 24} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad f(g(x)) &= f(x^2 - 1) \\ &= (x^2 - 1)^2 + 2(x^2 - 1) + 3 \\ &= x^4 - 2x^2 + 1 + 2x^2 - 2 + 3 \\ &= \boxed{x^4 + 2} \end{aligned}$$

## \* PART 4

$$\begin{array}{r} \textcircled{1} \quad x+1 \overline{) x^2 - 9x + 10} \\ \underline{x^2 - x} \phantom{+ 10} \\ -10x + 10 \\ \underline{+10x + 10} \\ 20 \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad 3x+1 \overline{) x^2 - 2x + 4} \\ \underline{3x^3 - 5x^2 + 10x - 3} \\ -3x^2 - x^2 \phantom{+ 10x - 3} \\ \underline{-6x^2 + 10x} \phantom{- 3} \\ +6x^2 + 2x \phantom{- 3} \\ \underline{12x - 3} \\ -12x - 4 \\ \hline -7 \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad 2x-5 \overline{) x^2 - 2x - 5} \\ \underline{2x^3 - 9x^2 + 0x + 15} \\ -2x^3 + 5x^2 \phantom{+ 0x + 15} \\ \underline{-4x^2 + 0x} \phantom{+ 15} \\ +4x^2 - 10x \phantom{+ 15} \\ \underline{-10x + 15} \\ +10x - 25 \\ \hline -10 \end{array}$$

continued

# \* PART 4 (continued)

④  $4x^2 - x - 7 + \frac{11x+5}{x^2+x+2}$

$$\begin{array}{r} x^2+x+2 \overline{) 4x^4+3x^3+0x^2+2x+1} \\ \underline{4x^4+4x^3+8x^2} \phantom{+1} \\ -x^3-8x^2+2x \phantom{+1} \\ \underline{+x^3+x^2+2x} \phantom{+1} \\ -7x^2+4x+1 \\ \underline{+7x^2+7x+14} \\ 11x+15 \end{array}$$

# \* PART 5

①  $(x^2+5x+6) \div (x-1)$   
 $\begin{matrix} x-1=0 \\ x=1 \end{matrix}$

$$\begin{array}{r} 1 \overline{) 1 \quad 5 \quad 6} \\ \underline{1 \quad 1 \quad 6} \\ 12 \end{array}$$

$$x+6 + \frac{12}{x-1}$$

②  $(6+a^3-a) \div (a+2)$   
 $\begin{matrix} 6a^3-a+6 & a+2=0 \\ & a=-2 \end{matrix}$

$$\begin{array}{r} -2 \overline{) 1 \quad 0 \quad -1 \quad 6} \\ \underline{-2 \quad 4 \quad -6} \\ 1 \quad -2 \quad 3 \quad 0 \end{array}$$

$$a^2-2a+3$$

typo, should be  $3x^3$

③  $(3x^3-2x^2+3x-4) \div (x-3)$   
 $\begin{matrix} x-3=0 \\ x=3 \end{matrix}$

$$\begin{array}{r} 3 \overline{) 3 \quad -2 \quad 3 \quad -4} \\ \underline{3 \quad 9 \quad 21 \quad 72} \\ 3 \quad -7 \quad 24 \quad 68 \end{array}$$

$$3x^2+7x+24 + \frac{68}{x-3}$$

If you did:

$$(3x^2-2x^2+3x-4) \div (x-3)$$

$$(x^2+3x-4) \div (x-3)$$

$$\begin{matrix} x-3=0 \\ x=3 \end{matrix}$$

$$\begin{array}{r} 3 \overline{) 1 \quad 3 \quad -4} \\ \underline{3 \quad 18} \\ 1 \quad 6 \quad 14 \end{array}$$

$$x+6 + \frac{14}{x-3}$$

# \*PART 6

$$\textcircled{1} \frac{2x^2+13x+20}{2x^2+17x+30}$$

$$\frac{(2x+5)(x+4)}{(2x+5)(x+6)}$$

$$\frac{x+4}{x+6}$$

$$\textcircled{2} \frac{x+3}{x^2-x-12}$$

$$\frac{x+3}{(x-4)(x+3)}$$

$$\frac{1}{x-4}$$

$$\textcircled{3} \frac{x^2-36}{6-x}$$

$$\frac{(x+6)(x-6)}{-1(x-6)}$$

$$\frac{x+6}{-1}$$

$$-x-6$$

# \*PART 7

$$\textcircled{1} \frac{x^2-25}{x^2+6x+9} \cdot \frac{2(x^2-9)}{x^2+10x+25}$$

$$\frac{(x+5)(x-5)}{(x+3)(x+3)} \cdot \frac{2(x+3)(x-3)}{(x+5)(x+5)}$$

$$\frac{2(x-5)(x-3)}{(x+3)(x+5)}$$

$$\frac{2(x^2-8x+15)}{x^2+8x+15}$$

$$\frac{2x^2-16x+30}{x^2+8x+15}$$

$$\textcircled{2} \frac{3xy^2}{2ab} \div \frac{9xy}{ab^2}$$

$$\frac{3xy^2}{2ab} \cdot \frac{ab^2}{9xy}$$

$$\frac{by}{6}$$

$$\textcircled{3} \left( \frac{4x^2-1}{x^3+1} \right) \div \left( \frac{2x-1}{x^2-1} \right)$$

$$\frac{4x^2-1}{x^3+1} \cdot \frac{x^2-1}{2x-1}$$

$$\frac{(2x+1)(2x-1)}{(x+1)(x^2-x+1)} \cdot \frac{(x+1)(x-1)}{2x-1}$$

$$\frac{(2x+1)(x-1)}{x^2-x+1} = \frac{2x^2-x-1}{x^2-x+1}$$



# PART 8

Pre-calc  
Summer  
Packet  
Solutions

$$\textcircled{1} \frac{1(x-1)}{(x+1)(x-1)} + \frac{2(x+1)}{(x-1)(x+1)}$$

$$\frac{x-1}{x^2-1} + \frac{2x+2}{x^2-1}$$

$$\frac{x-1+2x+2}{x^2-1}$$

$$\frac{3x+1}{x^2-1}$$

$$\textcircled{2} \frac{3(a+3)}{(a-2)(a+3)} - \frac{1(a-2)}{(a+3)(a-2)}$$

$$\frac{3a+9}{a^2+a-6} - \frac{(a-2)}{a^2+a-6}$$

$$\frac{3a+9-a+2}{a^2+a-6}$$

$$\frac{2a+11}{a^2+a-6}$$

$$\textcircled{3} \frac{2}{x^2} \cdot \frac{3}{3} + \frac{1}{3x} \cdot \frac{x}{x}$$

$$\frac{6}{3x^2} + \frac{x}{3x^2}$$

$$\frac{6+x}{3x^2}$$

$$\textcircled{4} \frac{x(x+1)}{x+1(x+1)} - \frac{1}{(x+1)^2}$$

$$\frac{x^2+x-1}{x^2+2x+1}$$

# \*PART 9

$$\textcircled{1} \left( \frac{\frac{1}{2} + \frac{2}{3}}{\frac{1}{4}} \right)$$

$$\left( \frac{1}{2} + \frac{2}{3} \right) \cdot \frac{4}{1}$$

$$2 + \frac{8}{3}$$

$$\boxed{\frac{14}{3} \text{ OR } 4 \frac{2}{3}}$$

$$\textcircled{2} \left( \frac{\frac{1}{x} + 2}{\frac{1}{x+1}} \right)$$

$$\left( \frac{1}{x} + 2 \right) \div \left( \frac{1}{x+1} \right)$$

$$\left( \frac{1}{x} + \frac{2 \cdot x}{1 \cdot x} \right) \cdot \frac{x+1}{1}$$

$$\frac{1+2x}{x} \cdot \frac{x+1}{1}$$

$$\frac{2x+1}{x} \cdot \frac{x+1}{1}$$

$$\boxed{\frac{2x^2+3x+1}{x}}$$

$$\textcircled{3} \left( \frac{\frac{1(x-3)}{x+3(x-3)} - \frac{1}{x^2-9}}{\frac{1}{x^2+6x+9}} \right)$$

$$\left( \frac{\frac{x-3}{x^2-9} - \frac{1}{x^2-9}}{\frac{1}{x^2+6x+9}} \right)$$

$$\frac{x-3-1}{x^2-9} \cdot \frac{x^2+6x+9}{1}$$

$$\frac{x-4}{(x+3)(x-3)} \cdot \frac{(x+3)(x+3)}{1}$$

$$\boxed{\frac{x^2-x-12}{x-3}}$$

# \* PART 10

Pre-Calc.  
Summer  
Packet  
Solutions

$$\begin{aligned} \textcircled{1} \quad \frac{1}{X-1} &= \frac{4}{X+2} \\ 1(X+2) &= 4(X-1) \\ \cancel{X}+2 &= 4X-\cancel{4} \\ \hline 2 &= 3X-\cancel{4} \\ +4 & \quad \quad \cancel{+4} \\ \hline 6 &= 3X \\ \frac{6}{3} &= \frac{3X}{3} \\ \boxed{2} &= X \end{aligned}$$

→ check

$$\begin{aligned} \frac{1}{2-1} &= \frac{4}{2+2} \\ \frac{1}{1} &= \frac{4}{4} \checkmark \\ \text{It works!} \end{aligned}$$

$$\textcircled{2} \quad \left( \frac{1}{X} + \frac{X}{3} \right) = \left( \frac{1}{6} \right)^{6X}$$

$$\begin{aligned} 6 + 2X^2 &= \cancel{X} \\ -X & \quad \quad \cancel{+X} \\ \hline 2X^2 - X + 6 &= 0 \end{aligned}$$

$$2X^2 - X + 6 = 0$$

$$a=2, b=-1, c=6$$

$$X = \frac{1 \pm \sqrt{1-4(2)(6)}}{2(2)}$$

$$X = \frac{1 \pm \sqrt{1-48}}{4}$$

$$X = \frac{1 \pm \sqrt{-47}}{4}$$

$$\boxed{X = \frac{1 \pm i\sqrt{47}}{4}}$$

continued

\* [PART 10 (continued)]

Pre  
Surv  
Pack  
Soluti

$$\textcircled{3} \quad \frac{3}{x+2} - \frac{5x(x+2)}{x} = \frac{1}{5x} \quad \frac{1(5x)(x+2)}{5x}$$

$$15x - 5x - 10 = x + 2$$

$$\frac{10x - 10}{-x} = \frac{x + 2}{-x}$$

$$\frac{9x - 10 = 2}{+10 \quad +10}$$

$$9x = 12$$

$$x = \frac{4}{3}$$

this value will never make the denominator equal to zero

check

$$\frac{3}{\frac{4}{3} + 2} - \frac{1}{\frac{4}{3}} = \frac{1}{5(\frac{4}{3})}$$

$$\textcircled{4} \quad \frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}$$

$$10 + 4x - 8 = 5x$$

$$2 + 4x = 5x$$

$$2 = x$$

$$2 = x$$

check

$$\frac{10}{2^2 - 2(2)} + \frac{4}{2} = \frac{5}{2 - 2}$$

$$\frac{10}{4 - 4} + 2 = \frac{5}{0}$$

$\frac{10}{0} + 2 = \frac{5}{0}$  } undefined, so  $x = 2$  is an extraneous solution

⇒ No solution



# \*PART 11

$$\textcircled{1} \sqrt[3]{-27x^3y^9}$$

$$\sqrt[3]{-27} \sqrt[3]{x^3} \sqrt[3]{y^9}$$

$$\boxed{-3xy^3}$$

$$\textcircled{2} \sqrt[5]{\frac{x^5}{32y^{10}}}$$

$$\frac{\sqrt[5]{x^5}}{\sqrt[5]{32y^{10}}}$$

$$\frac{\sqrt[5]{x^5}}{\sqrt[5]{32} \sqrt[5]{y^{10}}}$$

$$\boxed{\frac{x}{2y^2}}$$

# \*PART 12

$$\textcircled{1} y^{\frac{1}{3}} = \sqrt[3]{y}$$

$$\textcircled{2} 25^{\frac{3}{2}} = (\sqrt{25})^3$$

$$= 5^3$$

$$= \boxed{125}$$

$$\textcircled{3} 6x^{\frac{4}{3}} = 6\sqrt[3]{x^4}$$

$$= 6\sqrt[3]{x^3} \sqrt[3]{x}$$

$$= \boxed{6x\sqrt[3]{x}}$$

$$\textcircled{4} \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\textcircled{5} \sqrt[5]{m^3} = m^{\frac{3}{5}}$$

$$\textcircled{6} \sqrt[4]{k^6} = k^{\frac{6}{4}}$$

$$= \boxed{k^{\frac{3}{2}}}$$

# \*PART 13

$$\textcircled{1} \sqrt{20x^3y^4}$$

$$\sqrt{20} \sqrt{x^3} \sqrt{y^4}$$

$$\sqrt{4}\sqrt{5} \sqrt{x^2}\sqrt{x} y^2$$

$$2\sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2$$

$$\boxed{2xy^2\sqrt{5x}}$$

$$\textcircled{2} \sqrt[3]{16m^3np^4}$$

$$\sqrt[3]{16} \sqrt[3]{m^3} \sqrt[3]{n} \sqrt[3]{p^4}$$

$$\sqrt[3]{8} \sqrt[3]{2} \cdot m \cdot \sqrt[3]{n} \sqrt[3]{p^3} \sqrt[3]{p}$$

$$2\sqrt[3]{2} \cdot m \sqrt[3]{n} \cdot p \cdot \sqrt[3]{p}$$

$$\boxed{2mp\sqrt[3]{2np}}$$

$$\textcircled{3} \sqrt[4]{32ab^4c^6}$$

$$\sqrt[4]{32} \sqrt[4]{a} \sqrt[4]{b^4} \sqrt[4]{c^6}$$

$$\sqrt[4]{16} \sqrt[4]{2} \sqrt[4]{a} b \sqrt[4]{c^4} \sqrt[4]{c^2}$$

$$2\sqrt[4]{2} \cdot \sqrt[4]{a} \cdot b \cdot c \cdot \sqrt[4]{c^2}$$

$$\boxed{2bc\sqrt[4]{2ac^2}}$$

# \* PART 14

$$\textcircled{1} \sqrt[3]{ab} + 4\sqrt[3]{ab} - \sqrt{ab}$$

$$\boxed{5\sqrt[3]{ab} - \sqrt{ab}}$$

$$\textcircled{2} x\sqrt{20} + \sqrt{5x^2} - x\sqrt{5}$$

$$x\sqrt{4\sqrt{5}} + \sqrt{5}\sqrt{x^2} - x\sqrt{5}$$

$$x \cdot 2\sqrt{5} + x\sqrt{5} - x\sqrt{5}$$

$$\boxed{2x\sqrt{5}}$$

$$\textcircled{3} \sqrt[3]{16ab^3} - 2b\sqrt[3]{2a} + \sqrt[3]{27ab}$$

$$\sqrt[3]{16}\sqrt[3]{a}\sqrt[3]{b^3} - 2b\sqrt[3]{2a} + \sqrt[3]{27}\sqrt[3]{ab}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{2} \cdot \sqrt[3]{a} \cdot b - 2b\sqrt[3]{2a} + 3\sqrt[3]{ab}$$

$$2 \cdot \sqrt[3]{2} \cdot \sqrt[3]{a} \cdot b - 2b\sqrt[3]{2a} + 3\sqrt[3]{ab}$$

$$2b\sqrt[3]{2a} - 2b\sqrt[3]{2a} + 3\sqrt[3]{ab}$$

$$\boxed{3\sqrt[3]{ab}}$$

# PART 15

$$\textcircled{1} \quad \sqrt[3]{\frac{2}{3x}} \\ \frac{\sqrt[3]{2}}{\sqrt[3]{3x}} \cdot \frac{\sqrt[3]{9x^2}}{\sqrt[3]{9x^2}}$$

$$\frac{\sqrt[3]{18x^2}}{\sqrt[3]{27x^3}}$$

$$\boxed{\frac{\sqrt[3]{18x^2}}{3x}}$$

$$\textcircled{2} \quad \frac{5}{\sqrt{10x}} \cdot \frac{\sqrt{10x}}{\sqrt{10x}}$$

$$\frac{\cancel{5}\sqrt{10x}}{\cancel{10}x}$$

$$\boxed{\frac{\sqrt{10x}}{2x}}$$

$$\textcircled{3} \quad \sqrt[4]{\frac{4}{3ab^2}}$$

$$\frac{\sqrt[4]{4}}{\sqrt[4]{3ab^2}} \cdot \frac{\sqrt[4]{27a^3b^2}}{\sqrt[4]{27a^3b^2}}$$

$$\sqrt[4]{108a^3b^2}$$

$$\sqrt[4]{81a^4b^4}$$

$$\boxed{\sqrt[4]{108a^3b^2}}$$

# \*PART 14

$$\textcircled{1} \quad \frac{2}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$\boxed{\frac{2\sqrt{x}+6}{x-9}}$$

$$\textcircled{2} \quad \frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{\sqrt{6}-3}{2-3}$$

$$\frac{\sqrt{6}-3}{-1}$$

$$\boxed{-\sqrt{6}+3}$$

$$\textcircled{3} \quad \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$\boxed{\frac{a+2\sqrt{ab}+b}{a-b}}$$

Pre-calc  
Summer  
Packet  
Solutions



# \* PART 17

$$\textcircled{1} (\sqrt[3]{2x-1})^3 = (\sqrt[3]{x+3})^3$$

$$\begin{array}{r} 2x-1 = x+3 \\ -x \quad -x \end{array}$$

$$\begin{array}{r} x-1 = 3 \\ +1 \quad +1 \end{array}$$

$$\boxed{x=4}$$

$$\textcircled{2} \sqrt{3x-1} + 2 = 5$$

$$\begin{array}{r} \sqrt{3x-1} + 2 = 5 \\ -2 \quad -2 \end{array}$$

$$\begin{array}{r} \sqrt{3x-1} = 3 \\ +1 \quad +1 \end{array}$$

$$\begin{array}{r} 3x = 10 \\ 3 \quad 3 \end{array}$$

$$\boxed{x = \frac{10}{3}}$$

$$\textcircled{3} \sqrt{x+1} - \sqrt{2x} = 7$$

$$\begin{array}{r} \sqrt{x+1} - \sqrt{2x} = 7 \\ +\sqrt{2x} \quad +\sqrt{2x} \end{array}$$

$$\begin{array}{r} x+1 = 2x + 14\sqrt{2x} + 49 \\ -2x \quad -2x \end{array}$$

$$\begin{array}{r} -x+1 = 14\sqrt{2x} + 49 \\ -49 \quad -49 \end{array}$$

$$\begin{array}{r} -x-48 = 14\sqrt{2x} \\ 14 \quad 14 \end{array}$$

$$\left(\frac{-x-48}{14}\right)^2 = (\sqrt{2x})^2$$

$$+96 \left( \frac{x^2 + 96x + 2304}{+96} \right) = (2x)196$$

$$\rightarrow x^2 + 96x + 2304 = 392x$$

$$\boxed{x^2 - 296x + 2304 = 0}$$

$$x = \frac{296 \pm \sqrt{(-296)^2 - 4(1)(2304)}}{2(1)}$$

$$x = \frac{296 \pm \sqrt{87616 - 9216}}{2}$$

$$x = \frac{296 \pm \sqrt{78400}}{2}$$

$$\boxed{x = 288 \text{ or } x = 8}$$

you can factor  $\rightarrow$  factors of 2304

2304  
1, 2304  
2, 1152  
4, 576  
8, 288

SO  $x^2 - 296x + 2304 = 0$   
 $(x-8)(x-288) = 0$   
 $x-8=0, x-288=0$   
 $x=8, x=288$



$$\begin{aligned} \textcircled{1} \quad f(g(x)) &= f(x^2 - 2) \\ &= 2(x^2 - 2) - 1 \\ &= 2x^2 - 4 - 1 \\ &= 2x^2 - 5 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad g(f(x)) &= g(2x-1) \\ &= (2x-1)^2 - 2 \\ &= 4x^2 - 4x + 1 - 2 \\ &= \boxed{4x^2 - 4x - 1} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad f(f(x)) &= f(2x-1) \\ &= 2(2x-1) - 1 \\ &= 4x - 2 - 1 \\ &= \boxed{4x - 3} \end{aligned}$$

\* | PART 19

$$\begin{aligned} \textcircled{1} \quad f(x) &= 2x-1 \\ y &= 2x-1 \\ x &= \frac{y+1}{2} \\ \hline \frac{x+1}{2} &= \frac{y}{2} \\ \frac{x+1}{2} &= y \\ f^{-1}(x) &= \frac{x+1}{2} \end{aligned}$$

$$\textcircled{2} g(x) = \frac{x+3}{4}$$

$$y = \frac{x+3}{4}$$

$$4(x) = \left(\frac{y+3}{4}\right)4$$

$$4x = y + \cancel{3}$$

$$g^{-1}(x) = 4x - 3$$

③  $h(x) = \sqrt[3]{x-2}$

$$y = \sqrt[3]{x-2}$$

$$(x) = (\sqrt[3]{y-2})^3$$

$$\frac{x^3}{+2} = \frac{y-2}{+2}$$

$$x^3 + 2 = 4$$

$$h^{-1}(x) = x^3 + 2$$

④  $f(x) = \frac{1}{3}x + 2$

$$y = \frac{1}{3}x + 2$$

$$X = \frac{1}{3}y + 2$$

$$3(x-2) = \left(\frac{1}{3}y\right)^3$$

$$3x - 6 = y$$

$$f^{-1}(x) = 3x - 6$$

# PART 20

$$\textcircled{1} \quad 4^x = 64$$

$$(2^2)^x = 2^6$$

$$2^{2x} = 2^6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$\boxed{x = 3}$$

$$\textcircled{2} \quad 25 = 125^{2x}$$

$$5^2 = (5^3)^{2x}$$

$$5^2 = 5^{6x}$$

$$\frac{2}{6} = \frac{6x}{6}$$

$$\frac{1}{3} = x$$

$$\textcircled{3} \quad 5^x = 27$$

$$\log 5^x = \log 27$$

$$\frac{x \log 5}{\log 5} = \frac{\log 27}{\log 5}$$

$$\boxed{x \approx 2.05}$$

$$\textcircled{4} \quad 3^{x+1} = 7^x$$

$$\log 3^{x+1} = \log 7^x$$

$$(x+1) \log 3 = x \log 7$$

$$x \log 3 + \log 3 = x \log 7$$

$$-x \log 3 \quad -x \log 3$$

$$\log 3 = x \log 7 - x \log 3$$

$$\rightarrow \log 3 = x \frac{(\log 7 - \log 3)}{\log 7 - \log 3}$$

$$\boxed{x \approx 1.30}$$

# PART 21

$$\textcircled{1} \log x + \log 2x$$

$$\log(x \cdot 2x)$$

$$\boxed{\log 2x^2}$$

$$\textcircled{2} \log x^3 - \log x$$

$$\log\left(\frac{x^3}{x}\right)$$

$$\boxed{\log x^2}$$

$$\textcircled{3} 7^{\log_7 2x} = y$$

$$\log_7 y = \log_7 2x$$

$$y = 2x$$

$$\boxed{2x}$$

$$\textcircled{4} \ln e^{x+2}$$

$$\log_e e^{x+2}$$

$$\boxed{x+2}$$

$$\textcircled{5} \log_5 5^2$$

$$\boxed{2}$$

# PART 22

$$\textcircled{1} \log_b x^2 = \log_b (2x-1)$$

$$\begin{array}{r} x^2 = 2x-1 \\ -2x \quad -2x \\ \hline x^2 - 2x = -1 \\ +1 \quad +1 \\ \hline \end{array}$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1=0$$

$$\boxed{x=1}$$

double root

$$\textcircled{2} \log_2 x = 4$$

$$2^4 = x$$

$$\boxed{x=16}$$

$$\textcircled{3} \log_2 8 = x$$

$$2^x = 8$$

$$2^x = 2^3$$

$$\boxed{x=3}$$

$$\textcircled{4} \log_2 x + \log_2 (x-2) = 3$$

$$\log_2 (x(x-2)) = 3$$

$$\log_2 (x^2 - 2x) = 3$$

$$2^3 = x^2 - 2x$$

$$\begin{array}{r} 8 \cancel{-} x^2 - 2x \\ -8 \\ \hline \end{array}$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$x-4=0, x+2=0$$

$$\begin{array}{l} x=4 \\ x=-2 \end{array}$$

# PART 23

$$\textcircled{1} (3+2i)-(2-i)+(4+3i)$$

$$3+2i-2+i+4+3i$$

$$\boxed{5+6i}$$

$$\textcircled{2} 3i(2+4i)$$

$$6i+12i^2$$

$$6i+12(-1)$$

$$6i-12$$

$$\boxed{-12+6i}$$

$$\textcircled{3} (3-i)(2+4i)$$

$$6+12i-2i-4i^2$$

$$6+10i-4(-1)$$

$$6+10i+4$$

$$\boxed{10+10i}$$